Labyrinthine granular landscapes

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We have numerically studied a model of granular landscape eroded by wind. We show the appearance of labyrinthic patterns when the wind orientation turns by 90°. The occurence of such structures is discussed. Morever, we introduce the density n_k of ''defects'' as the dynamic parameter governing the landscape evolution. A power-law behavior of n_k is found as a function of time. In the case of wind variations, the exponent (drastically) shifts from two to one. The presence of two asymptotic values of n_k implies the irreversibility of the labyrinthic formation process.

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I. INTRODUCTION

The ripple formation due to the wind blowing across a sand bed [1] has recently received much attention in the statistical physics community [2,3]. Indeed, the physical mechanisms involved are complex phenomena of granular transport. Experimental works, as well as natural observations [1], have underlined the primary role played by saltation in the emergence of ripples and dunes. Along this line, various models for ripple formation have been proposed in the past. Theoretical models that consider hopping and rolling grains have generally led to traveling ripple structures [2]. Simulations [3–5] have also considered various additional effects such as the screening of crests, grain reptation, and the existence of a grain ejection threshold. Among others, the Nishimori-Ouchi (NO) model [4] is able to reproduce a wide variety of different eolian structures: transverse, barkhanic, starlike, etc. The main advantage of such a numerical model is that all parameters can be easily tuned and the physical mechanisms can be deeply investigated.

In 1991, Goossens [6] performed an original experiment that is the following. In a wind tunnel, a 12×12 cm^2 rough granular landscape (dust size ≈ 32 μm) is eroded by an air flow ($v\approx 132$ cm s^{-1}). This created small ripples perpendicular to the wind direction. After 45 minutes, the wind orientation was changed by 90° and this produced diagonal structures instead of a new set of ripples perpendicular to the previous ones. The Goossens's experiment represents a good test for the NO model. In the present paper, we report simulations of this particular kind of landscape. This allows us to discuss the dynamics of this unusual phenomenon.

II. MODEL

In the Nishimori-Ouchi model [4], two kinds of granular transport processes are considered: (i) the saltation and, (ii) the potential-energy relaxation. The temporal evolution equation of the height of sand $h(x)$ at point of coordinate x reads

$$\frac{\partial h(x,t)}{\partial t}=A\left(N(\ell)\frac{d\ell}{dx}-N(x)\right)+D\frac{\partial^2 h(x,t)}{\partial x^2}. \qquad (1)$$

At the right-hand side of this equation, the first term represents the saltation process. Due to wind shear stress, grains are moved from a position $\ell(x)$ to a position x. The mean amplitude of the path length is given by the constant A. On the other side, the constant D is a relaxation coefficient. This second term takes in account the transport phenomena along the slopes of the surface, e.g., reptation and avalanches.

The NO model has been implemented as follows. A two-dimensional square lattice with periodic boundary conditions is considered. To each site i,j of the lattice is associated a real number $h_{i,j}$ that represents the height of the granular landscape at that position. Assume that the wind blows along the i axis. At each discrete time t, a site i,j is randomly choosen and a quantity $q_{i,j}$ of matter is displaced by saltation from this site towards the site $i+\ell_{i,j},j$ that is incremented by the height $q_{i,j}$. Both quantities $\ell_{i,j}$ and $q_{i,j}$ are determined by

$$\ell_{i,j}=\alpha(\tanh\nabla h_{i,j}+1),$$
$$q_{i,j}=\beta(1+\epsilon-\tanh\nabla h_{i,j}), \qquad (2)$$

where α and β are dynamical constants and the parameter ϵ is the minimum quantity of sand that is displaced by saltation. The mathematical form ($\tanh\nabla h$) of those relationships assumes that the local slope mainly controls the granular transport. The flux of sand extracted from the faces exposed to the wind is indeed smaller than that screened by crests. After the saltation process (2), a relaxation of the landscape is assumed (creeping and avalanches) before the next time step $t+1$ takes place. The relaxation reads

$$h_{i,j}(t+1)=h_{i,j}(t)+D\left[\frac{1}{6}\sum_{nn}h_{nn}(t)+\frac{1}{12}\sum_{nnn}h_{nnn}(t)-h_{i,j}(t)\right],$$

$$h_{i+\ell_{i,j},j}(t+1)=h_{i+\ell_{i,j},j}(t)-D\left[\frac{1}{6}\sum_{nn}h_{nn}(t)+\frac{1}{12}\sum_{nnn}h_{nnn}(t)-h_{i+\ell_{i,j},j}(t)\right],$$

where the summations run over nearest neighbors (nn) and next nearest neighbors (nnn) of both sites i,j and $i+\ell_{i,j},j$. This equation is the discrete counterpart of the La-

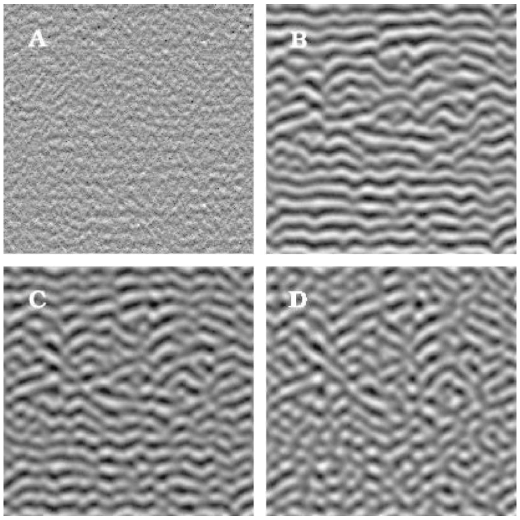

FIG. 1. Four different stages of a granular landscape evolution within the NO model. When the wind direction changes, a labyrinthic pattern appears. The simulation parameters are: $\alpha=2.5$, $\beta=5$, $D=0.4$, $\epsilon=0.3$. The lattice size is 101×101. (a) t=0.2 a.u. and wind direction is up, (b) t=0.48 a.u. and wind direction is up, (c) t=0.52 a.u. and wind direction is right, (d) t=1 a.u. and wind direction is right.

placian relaxation of Eq. (1). The process is repeated a large number of times. Typically, we stop the simulation after $t=2.5\times10^7$ steps on 201×201 lattices. We intentionally choose a lattice size that is not commensurable with the mean saltation lenght α.

III. RESULTS AND DISCUSSION

We have performed extensive simulations by varying all the parameters: α, β, D, and ϵ. Modifying α changes the mean ripple wavelength (the distance between two sucessive crests), while D affects the aspect ratio (amplitude) of the ripples. The values taken by β and ϵ permit or not the appearence of ripples: typicaly $\beta\in[0.2,0.6]$ and $\epsilon=0.3$. Note that we have normalized the time t by the duration of the simulation, involving $t\in[0,1]$ in arbitrary units (a.u.).

Figure 1 shows a typical result of our simulations for $\alpha=2.5$, $\beta=5$, $D=0.4$, $\epsilon=0.3$. The granular landscape is shown for four different stages of evolution. One observes on the top row the early formation of ripples perpendicular to the wind direction. On the bottom row, the wind direction has changed by 90° clockwise and a labyrinthic structure appears. This observation emphasizes the impact of the initial topography on the orientation of the ripples crests. One should note that such labyrinthic structures are strikingly similar to Goossens' ones [6].

In order to quantify the effect(s) of wind variations, we have measured the maximum ripple amplitude A_{max} for both constant and variable winds. This quantity was also experimentally measured in [6]. As the wind orientation is changed by 90°, no significant change of A_{max} is observed, similar to experiments [6]. This means that a brutal change in the wind direction does not modify the net deposit of sediments on the crests. The competition between transport and deposit phe-

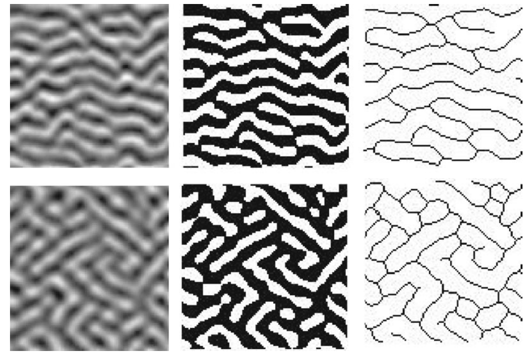

FIG. 2. Illustration of the procedure of kink counting in the case of constant wind (top row) and variable wind (bottom row). Grayscale images are created with crests in black (left). The binary (black/white) images are extracted and show the crests (middle). An iterative erosion technique leads to a skeleton (right).

nomena is not deteriorated by the perturbation of the wind orientation. Actually, A_{max} represents adequately this competition, but is not a relevant dynamical parameter in order to understand the formation of labyrinthic structures.

Looking for details in Fig. 1, one can observe that: (i) diagonal structures appear at the vicinity of ''defects'' of the primary landscape. Those ''defects'' are *kinks* and *antikinks*. A kink is a bifurcation of a crest, while an antikink is a termination of a crest, i.e., a bifurcation of the valleys. Moreover, kinks and antikinks are not independent at all: they are formed by pairs. Kinks and antikinks may be considered as nucleation centers for new ripples when the wind direction changes. One understands the formation of labyrinths as follows. Old ripples are pushed in the new wind direction. If their crests are perpendicular to the new direction, ripples are compressed. However, near a ''defect,'' the angle between the crest and the wind is smaller than 90°. A rotation of the crest is thus initialized there. This leads to diagonal structures. (ii) The formation of a labyrinthiclike structure involves the growth of the number of ''defects.''

Let us consider the relevant parameter: the density n_k of kinks. This quantity is defined as the number of kinks present on the surface, divided by the area of the lattice. In order to measure the number of kinks present in the landscape, we proceeded as follows (see the illustration in Fig. 2). The surface is recorded in grayscale images at different stages of evolution. The darkness indicates the height of sand, i.e., crests are in black while valleys are in white. Images are then analyzed using common tools of image analysis. First, a threshold is applied in order to get binary (black/white) images with crests in black. Then, a function reduces all crests to a skeleton through an iterative erosion technique. The last step concerns the countdown itself. The program browses the skeleton line by line. When a black point is met (a crest), the number of its black neighbors is counted. If this number is greater or equal to three, the point is necessary a kink. One should note that this method can be applied to images of real experiments.

Figure 3 presents the temporal evolution of n_k in both cases: constant and variable wind. One can see that without any wind modification, n_k decreases as a function of time

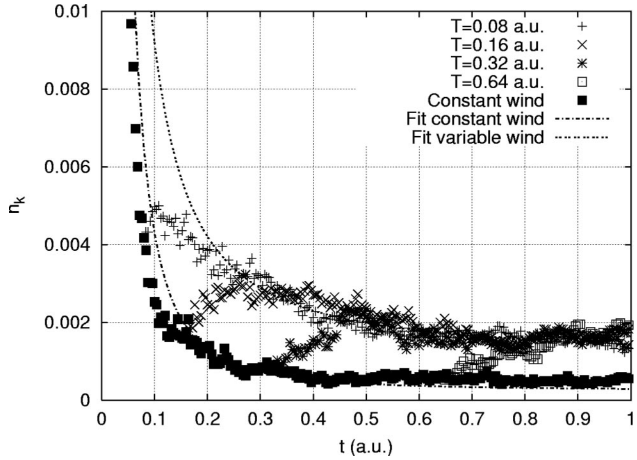

FIG. 3. The density n_k of kinks as a function of time t. The simulation parameters are : $\alpha = 2.5$, $\beta = 5$, $D = 0.4$, $\epsilon = 0.3$, the lattice size is 201×201. Different times T for wind orientation changes are illustrated. The curves are fits for both cases: constant wind (bottom curve) and variable wind (top curve).

(black squares). When a wind change occurs at time T, the density of kinks suddenly increases. In Fig. 3, different wind variations are illustrated for different times T. After the jump of n_k observed at time T, the kink density continues to decreases slowly.

Since the kink density decreases in a faster way than a logarithmiclike law, and in a slower way than an exponential one, we have assumed a power-law decay

$$n_k(t) = a + \frac{b}{c + t^d}, \tag{3}$$

where a, b, c, and d are fitting parameters. The parameter a represents the asymptotic density of kinks, while the exponent d captures the dynamics of the landscape. Indeed, a large value of d involves a fast decrease of the kink density; such a situation implies that the surface can be easily modified by the wind. On the other hand, a small value of d means that the patterns are less affected by the variation of the wind orientation. Looking for details in Fig. 3, one should note that the wind variation does not affect the decay law of n_k after the jump. Actually, the density of kinks follows Eq. (3) before and after the wind change.

Typical fits using Eq. (3) are drawn in Fig. 3 and parameters are reported in Table I. The upper line of Fig. 3 correspond to the case of a variable wind, and is characterized by $d \approx 1$, while the lower curve is for a constant wind and follows $d \approx 2$. This difference implies a greater stability of the labyrinthic structure, and the existence of two modes.

TABLE I. Parameters a and d of Eq. (3) fitted for both constant and variable winds.

	a	d
Constant wind	$2.45 \times 10^{-4} \pm 7.6 \times 10^{-5}$	1.98 ± 0.03
Variable wind	$9.68 \times 10^{-4} \pm 1 \times 10^{-4}$	1.16 ± 0.09

An interesting observation is that if a second wind change occurs on the labyrinthic structure, n_k is not affected. Once the diagonal structure is created, any come back to the transversal one is prohibited. The process is irreversible. However, if the initial topography is composed by ripples with a small amount of defects, the landscape can evolve to a nearly transversal structure. This behavior comes from the lack of kinks. Indeed, if n_k is initially small, a few number of labyrinths will be formed. As a consequence, the landscape is less stable.

Moreover, the formation of labyrinthic structures induces a kind of ‘‘memory effect.’’ Indeed, asymptotic values a listed in Table I are significantly different if one compares constant and variable cases. A difference in asymptotic values is a strong result supporting the idea that there is a memory of the wind direction on the landscape evolution. After a wind change, the evolution of the landscape depends essentially on the former topography. Even after a long time, the surface always evolves in a way depending on its history, i.e., on the number on wind orientation changes. The question is to know if real granular landscapes show this memory effect. This is left for future experimental work.

IV. SUMMARY

In summary, we have simulated unusual labyrinthic landscapes observed in earlier experiments. We have investigated the formation and evolution of these landscapes. We have demonstrated that the density of defects in a ripple structure is a relevant parameter to characterize the temporal evolution of such structures. Indeed, the number of ‘‘defects’’ present in the landscape decreases according to a negative power law of time. If wind orientation is changed, the power exponent shifts from a value two to the value one. These exponents do not dependent on the occurence of wind change. We have also shown the emergence of a memory effect in the asymptotic value of the kink density.

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